

Please check the examination details below before entering your candidate information

Candidate surname		Other names	
Pearson Edexcel International GCSE		Centre Number 	Candidate Number
Thursday 6 June 2019			
Morning (Time: 2 hours 30 minutes)		Paper Reference 4MB1/02	
Mathematics B Paper 2			
You must have: Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.			Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- **Calculators may be used.**

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Without sufficient working, correct answers may be awarded no marks.

Turn over ►

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Answer ALL ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 Malcolm is investigating the ages of the 200 members of a theatre company.

The table below gives information about the age group of each member of the company.

Age (t years)	Frequency
$10 < t \leq 20$	23
$20 < t \leq 30$	47
$30 < t \leq 40$	56
$40 < t \leq 50$	48
$50 < t \leq 60$	26

Malcolm is going to draw a pie chart for this information.

- (a) Calculate the size, in degrees, of the angle of the sector in the pie chart for those members of the company whose age group is $40 < t \leq 50$

(2)

- (b) Calculate an estimate for the mean age, to the nearest year, of the members of the theatre company.

(4)

(a) You have to find the ratio of those in age group $40 < t \leq 50$ to the total and then find an angle to to 360° .

$$48 : 200$$

$$\times \frac{48}{200}$$

$$86.4 : 360$$

$$\times \frac{48}{200}$$

$\therefore 86.4^\circ$

(b) For the mean age, you have to find the sum of the midpoint of age intervals multiplied by frequency divided by the total frequency.

$$\frac{(15 \times 23) + (25 \times 47) + (35 \times 56) + (45 \times 48) + (55 \times 26)}{200} = 35.35$$

≈ 35 years



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Question 1 continued

Handwriting practice area with horizontal dotted lines.

(Total for Question 1 is 6 marks)



- 2 In 2017, country A had a population of 2.35×10^7 people.

Of these people, 48% were male.

- (a) Calculate the number of males in country A in 2017

(2)

Country A is divided into three regions. These three regions are called East Region, Central Region and West Region.

In 2017, the ratio of the number of males in the East Region to the number of males in the Central Region to the number of males in the West Region was 5 : 3 : 2

- (b) Calculate the number of males in the Central Region in 2017
Give your answer in standard form.

(2)

In 2017, the number of females in the Central Region was 12.5% greater than the number of males in the Central Region.

- (c) Calculate the number of females in the Central Region in 2017

(2)

In 2010, country B had a population of 2.5×10^7 people.

From 2010 to 2014, the population of country B increased by 2.4%
From 2014 to 2018, the population of country B decreased by 2.4%

- (d) Calculate the population of country B in 2018

(2)

- (a) Find 48% of 2.35×10^7

$$(2.35 \times 10^7) \times \frac{48}{100} = 11280000$$

$$\approx 1.128 \times 10^7 \text{ males}$$

- (b) In 2017, there were 1.128×10^7 males who are distributed in the ratio 5 : 3 : 2

$$\frac{1.128 \times 10^7}{5+3+2} = 1.128 \times 10^6 \rightarrow 1 \text{ part represent } 1.128 \times 10^6 \text{ males}$$

East Region : Central Region : West Region

5 : 3 : 2

5.54×10^6 : 3.384×10^6 : 2.256×10^6 $\times 1.128 \times 10^6$

There were 3.384×10^6 males in Central Region.



Question 2 continued

(c) First calculate 12.5% of 3.384×10^6 and add to 3.384×10^6 for number of females

$$\frac{12.5}{100} \times 3.384 \times 10^6 = 4.23 \times 10^5$$

$$3.384 \times 10^6 + 4.23 \times 10^5 = 3.807 \times 10^6 \text{ females}$$

Alternatively: Since the population of females is 100% of the male population plus 12.5%, it will be 112.5%.

$$\frac{112.5}{100} \times 3.384 \times 10^6 = 3.807 \times 10^6 \text{ females}$$

(d) First calculate 2.4% of 2.5×10^7 and add to 2.5×10^7 for the population in 2014.

$$\frac{2.4}{100} \times 2.5 \times 10^7 = 6 \times 10^5$$

$$2.5 \times 10^7 + 6 \times 10^5 = 2.56 \times 10^7$$

Now find 2.4% of 2.56×10^7 and subtract.

$$\frac{2.4}{100} \times 2.56 \times 10^7 = 6.144 \times 10^5$$

$$2.56 \times 10^7 - 6.144 \times 10^5 = 2.49856 \times 10^7$$

(Total for Question 2 is 8 marks)



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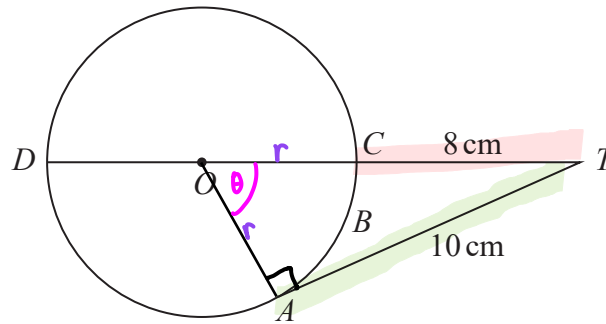


Figure 1

Figure 1 shows the circle $ABCD$ with centre O and diameter DC .

The point T is such that $TCOD$ is a straight line and TA is the tangent to the circle at A .

$$AT = 10 \text{ cm}$$

$$TC = 8 \text{ cm}$$

(a) Calculate the radius, in cm, of the circle.

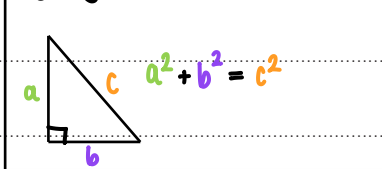
(3)

(b) Calculate the length, in cm to 3 significant figures, of the arc ABC .

(3)

(a) The tangent to a circle is perpendicular so $\triangle TOA$ is a right-angled triangle and we can use Pythagoras.

Pythagoras's theorem:

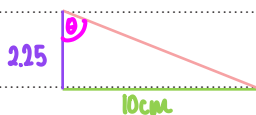


$$\begin{aligned} r^2 + 10^2 &= (r + 8)^2 \\ r^2 + 100 &= r^2 + 16r + 64 \quad \text{Expand} \\ -r^2 \quad -64 & \quad -r^2 \\ 100 &= 16r + 64 \\ -64 \quad -64 & \quad -64 \\ 36 &= 16r \\ \div 16 \quad \div 16 & \quad \div 16 \\ 2.25 \text{ cm} &= r \end{aligned}$$

(b) The equation for the length of an arc is: $\frac{\theta}{360} \times \pi d$ where θ is the angle of the arc.

To find θ we can use $\angle HTA$.

Since we have the opposite and adjacent we can use $\tan^{-1}$.



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \theta = \frac{10}{2.25}$$

$$\theta = \tan^{-1} \left(\frac{10}{2.25} \right)$$



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Question 3 continued

$$\theta = 77.31\dots$$

$$\text{Length of arc ABC: } \frac{77.31\dots}{360} \times \pi \times 2(2.25) = 3.0363\dots$$
$$\approx 3.04 \text{ cm}$$

(Total for Question 3 is 6 marks)



P 6 0 2 5 7 A 0 7 3 6

- 4 Triangle A is drawn on the grid opposite.

The points $(1, 2)$, $(3, -2)$ and $(1, 4)$ are the vertices of triangle B .

- (a) On the grid, draw and label triangle B .

(1)

- (b) Describe fully the single transformation that maps triangle A onto triangle B .

(2)

Triangle C is the image of triangle B under an enlargement with scale factor 2 and centre of enlargement the origin.

- (c) On the grid, draw and label triangle C .

(3)

Triangle D is the image of triangle C under the transformation with matrix $\mathbf{M}$ where

$$\mathbf{M} = \begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix}$$

- (d) On the grid, draw and label triangle D .

(3)

Triangle C is the image of triangle D under the transformation with matrix $\mathbf{N}$.

- (e) Find the matrix $\mathbf{N}$.

(3)

(b) Triangle A is transformed to triangle B by translation of vector $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

(c) $\begin{pmatrix} 1 \\ 2 \end{pmatrix} \times 2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} \times 2 = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} \times 2 = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

(d) Rule for multiplying a 1×2 by a 2×2 matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}(2) + 0(4) \\ \frac{1}{2}(2) + -1(4) \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

$$\left[\text{The inverse of matrix } \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ is } \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \right]$$

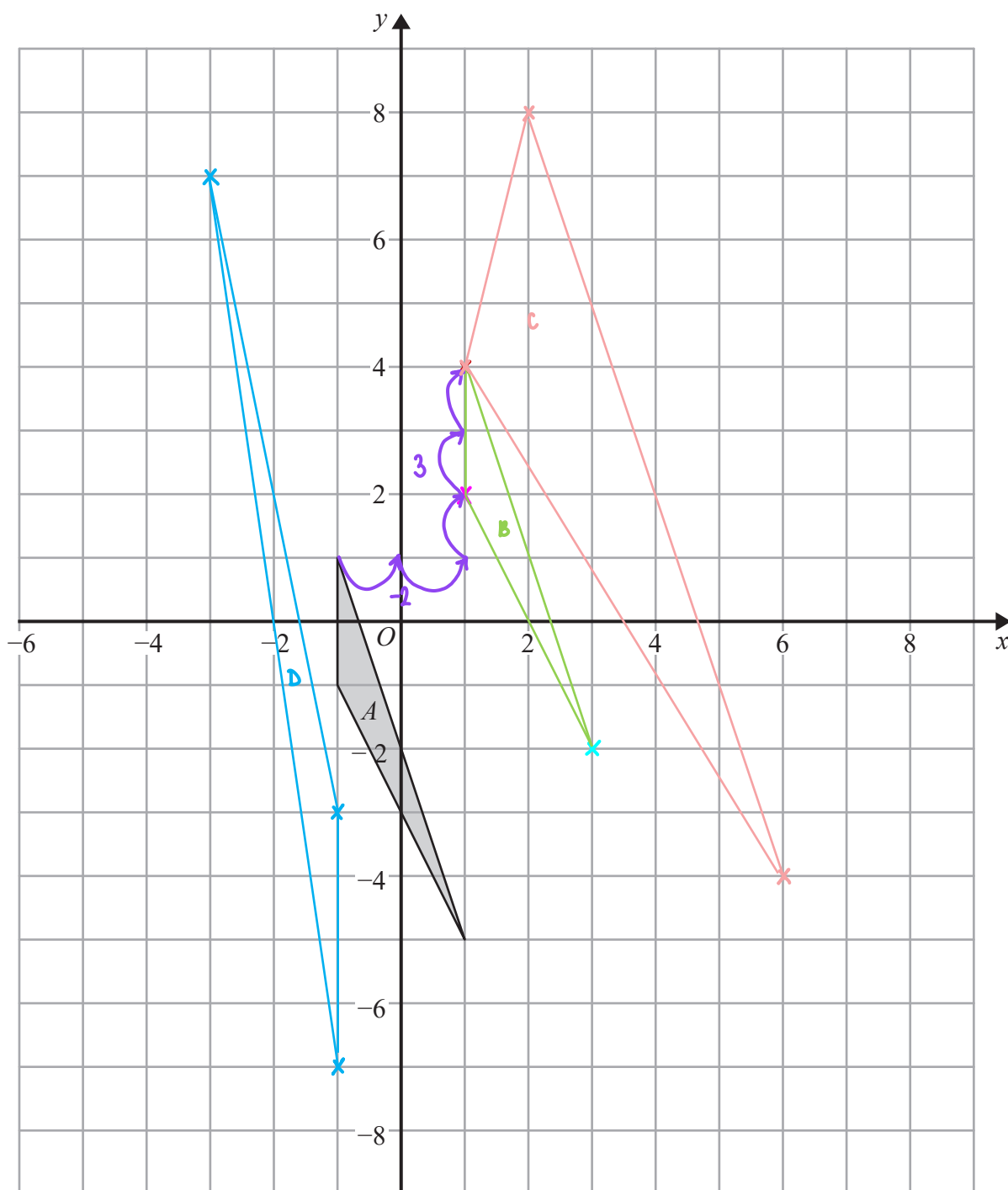


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Question 4 continued



Turn over for a spare grid if you need to redraw your triangles.



Question 4 continued

$$\begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix} \times \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}(6) + 0(-4) \\ \frac{1}{2}(6) + -1(-4) \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 8 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}(2) + 0(8) \\ \frac{1}{2}(2) + -1(8) \end{pmatrix} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}$$

(e) Matrix N is the inverse of matrix M.

$$\left[\text{The inverse of matrix } \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ is } \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \right]$$

$$\begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & -1 \end{pmatrix}^{-1} = \frac{1}{(-\frac{1}{2})(-1) - (0)(\frac{1}{2})} \begin{pmatrix} (-1) & -(0) \\ -(\frac{1}{2}) & (-\frac{1}{2}) \end{pmatrix}$$

$$= \frac{1}{\frac{1}{2}} \begin{pmatrix} -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$$= 2 \begin{pmatrix} -1 & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$



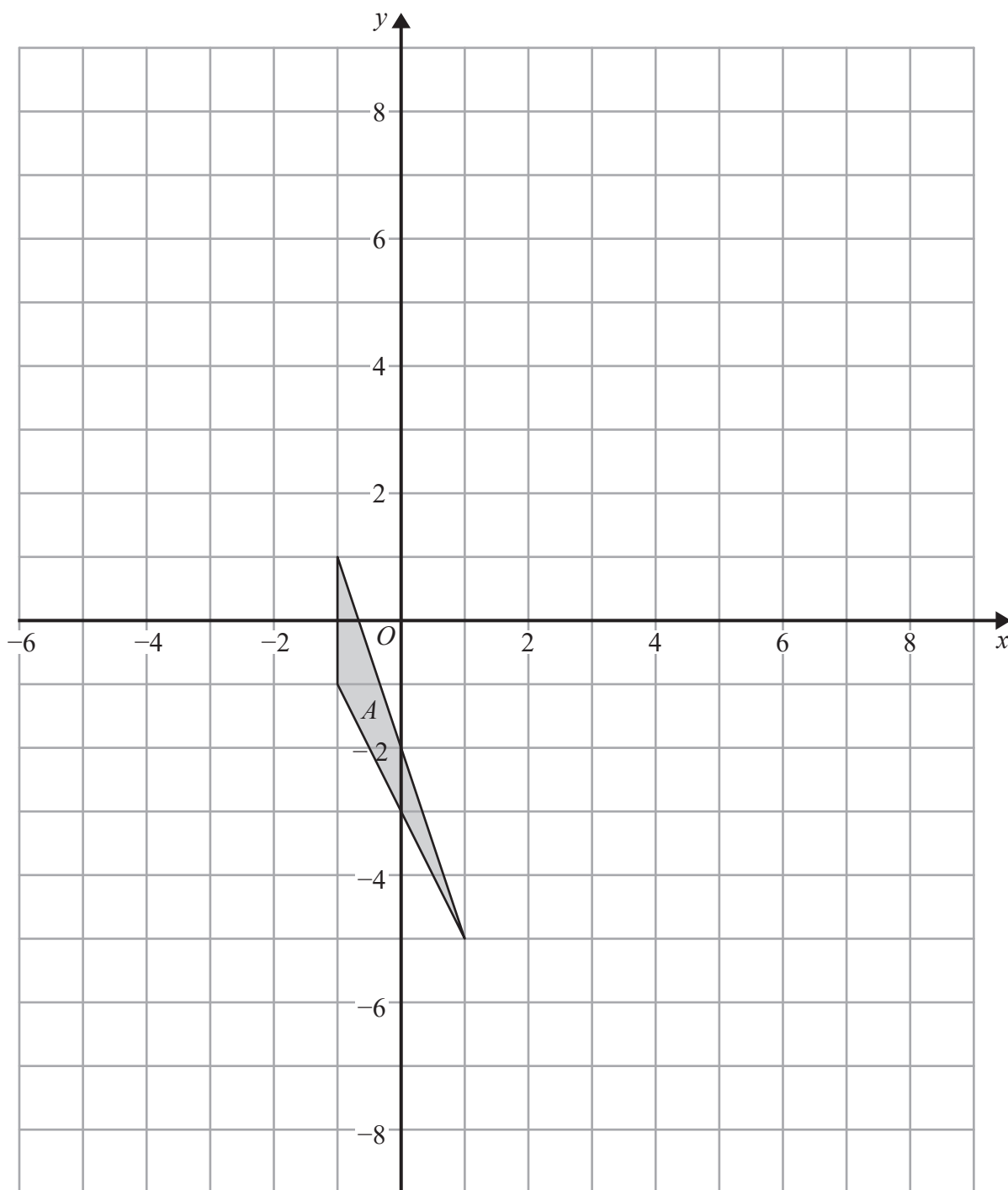
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Question 4 continued

Only use this grid if you need to redraw your triangles.



(Total for Question 4 is 12 marks)



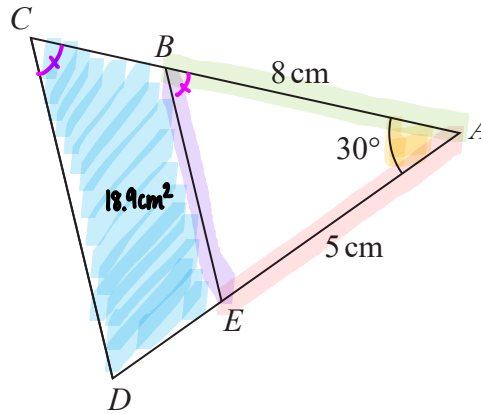


Diagram **NOT**
accurately drawn

Figure 2

Figure 2 shows the triangles ABE and ACD where B lies on AC and E lies on AD . The triangles are similar with $\angle ABE = \angle ACD$.

$$AB = 8 \text{ cm} \quad AE = 5 \text{ cm} \quad \angle BAE = 30^\circ$$

(a) Calculate the length, in cm to 3 significant figures, of BE . (3)

(b) Calculate the size, in degrees to 3 significant figures, of $\angle ABE$. (3)

The area of quadrilateral $BCDE$ is 18.9 cm^2

(c) Calculate the length, in cm to 3 significant figures, of CD . (4)

(a) You can use cosine to find BE .

$$a^2 = b^2 + c^2 - 2bc \cos A$$

where $a = BE$

$$(BE)^2 = (8)^2 + (5)^2 - 2(8)(5) \cos(30)$$

$b = 8$

$$BE = \sqrt{(8)^2 + (5)^2 - 2(8)(5) \cos(30)}$$

$c = 5$

$$= 4.440...$$

$A = 30^\circ$

$$\approx 4.44 \text{ cm}$$

$$\left[\begin{array}{l} \text{Cosine rule : } a^2 = b^2 + c^2 - 2bc \cos A \\ \text{Sine rule : } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \\ \text{Area of triangle} = \frac{1}{2} ab \sin C \end{array} \right]$$



Question 5 continued

(b) you can use sine rule to find $\angle ABE$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

where $a = 5$

$$\frac{\sin \angle ABE}{5} = \frac{\sin 30}{4.440...}$$

$A = \angle ABE$

$$\sin \angle ABE = \frac{\sin 30}{4.440...} \times 5$$

$b = 4.440...$

$B = 30$

$$\angle ABE = \sin^{-1} \left(\frac{\sin 30}{4.440...} \times 5 \right)$$

$$\angle ABE = 34.26...$$

$$\approx 34.3^\circ$$

(c) The two triangles are similar so if you can find the area scale factor you can find the length scale factor.

Area of triangle ABE:

$$\frac{1}{2} ab \sin C$$

$$\text{where } a = 8 \quad ABE = \frac{1}{2} \times (8) \times (5) \times \sin(30)$$

$$b = 5 \quad = 10 \text{ cm}^2$$

$$C = 30$$

Area of triangle ACD = triangle ABE + quadrilateral BCDE

$$ACD = 10 + 18.9$$

$$ACD = 28.9 \text{ cm}^2$$



$$ABE : ACD$$

$$\times \frac{28.9}{10}$$

$$10 : 28.9$$

$$BE : CD$$

$$4.440... : 7.548... \approx 7.55 \text{ cm}$$

$$\times \sqrt{\frac{28.9}{10}}$$



Question 5 continued

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(Total for Question 5 is 10 marks)



6

$$A = \{a, b, c, d\}$$

Write down all the subsets of A that contain the element b .

Here, you should list all the possible combinations that have b in them.

a	b	c	d
a	b	c	d
a	b	c	d
a	b	c	d
a	b	c	d
a	b	c	d
a	b	c	d
a	b	c	d

There are 8 possible sets: $\{b\}$, $\{b, c\}$, $\{b, d\}$, $\{a, b\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{b, c, d\}$, $\{a, b, c, d\}$

(Total for Question 6 is 4 marks)



- 7 To pass a course, Preety has to pass two tests, test **A** and test **B**.

Each test only has to be passed once.

Passing test **A** and passing test **B** are independent events.

Preety has three attempts to pass test **A**.

The probability that she passes test **A** on her first attempt is $\frac{3}{5}$

If she fails on her first attempt, the probability that she passes on her second attempt is $\frac{2}{5}$

If she fails on her second attempt, the probability that she passes on her third attempt is $\frac{3}{8}$

- (a) Calculate the probability she passes test **A**.

(3)

Preety has two attempts to pass test **B**.

The probability that she passes test **B** on her first attempt is $\frac{3}{5}$

The probability that she passes test **B** is $\frac{3}{4}$

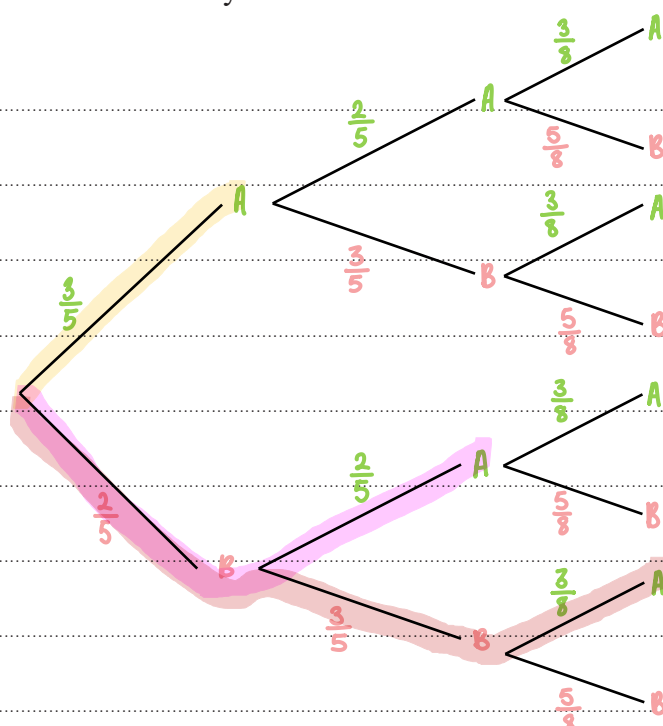
- (b) Calculate the probability that Preety passes test **B** on her second attempt, given that she fails test **B** on her first attempt.

(3)

Preety decides to take the two tests only if the probability that she passes the course is greater than 0.5

- (c) State whether or not Preety should take the two tests.
Give a reason for your answer.

(2)



Question 7 continued

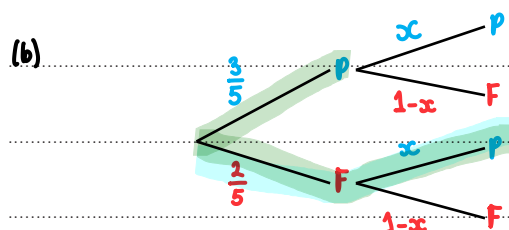
(a) To calculate the probability of passing test A, you have to calculate the probability of passing on the first, second and third attempt.

Passing on the first attempt: $\frac{3}{5}$

Passing on the second attempt: $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

Passing on the third attempt: $\frac{2}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{18}{200}$

$$P(\text{passing A}) = \frac{3}{5} + \frac{4}{25} + \frac{18}{200} = \frac{17}{20}$$



Here we need to find the probability of passing on the second attempt using the information given about the total probability of passing.

$$P(\text{passing B}) = \frac{3}{4}$$

$$P(\text{passing B}) = \frac{3}{5} + \left(\frac{2}{5} \times x\right) = \frac{3}{5} + \frac{2x}{5}$$

$$= \frac{3+2x}{5}$$

$$\begin{aligned} \frac{3+2x}{5} &= \frac{3}{4} \\ \times 5 & \quad \times 5 \\ 3+2x &= \frac{15}{4} \\ -3 & \quad -3 \\ 2x &= \frac{3}{4} \\ \div 2 & \quad \div 2 \\ x &= \frac{3}{8} \end{aligned}$$

Probability of failing on first attempt and passing on second: $\frac{2}{5} \times \frac{3}{8} = \frac{6}{40}$

$$= \frac{3}{20}$$



Question 7 continued

(c) You can use the probabilities you found on the two previous questions to calculate the total probability of passing.

$$P(\text{passing both}) = P(\text{passing A}) \times P(\text{passing B})$$

$$= \frac{17}{20} \times \frac{3}{4}$$

$$= \frac{51}{80}$$

$$= 0.6375$$

$$0.6375 > 0.5$$

$\therefore$ She should take the test.



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Question 7 continued

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(Total for Question 7 is 8 marks)



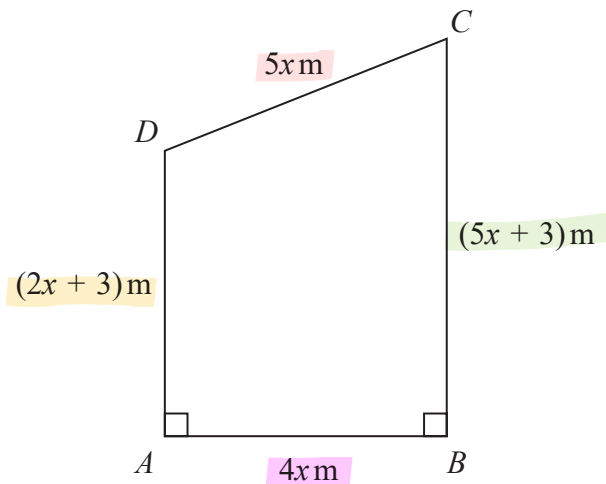


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Figure 3

Figure 3 shows the plan for a lawn that is in the shape of a trapezium $ABCD$ in which

$$AB = 4x \text{ metres} \quad BC = (5x + 3) \text{ metres} \quad CD = 5x \text{ metres} \quad DA = (2x + 3) \text{ metres}$$

The perimeter of the lawn is P metres.

- (a) Find and simplify an expression for P in terms of x .

(2)

The area of the lawn is $A \text{ m}^2$

- (b) Show that $A = 14x^2 + 12x$

(2)

The owner of the lawn wants the perimeter of the lawn to be greater than 52 m.
He also wants the area of the lawn to be at most 162 m^2

- (c) Find the range of possible values of x .
Show clear algebraic working.

(6)

(a) The perimeter is the sum of all sides.

$$P = (4x) + (5x+3) + (5x) + (2x+3)$$

$$= 16x + 6$$

$$\left[\text{Area of trapezium} = \frac{1}{2} (a+b)h \right]$$



Question 8 continued

(b) Area of a trapezium = $\frac{1}{2}(a+b)h$

$$\text{where } a = 2x+3 \quad A = \frac{1}{2}((2x+3) + (5x+3)) \times 4x$$

$$b = 5x+3 \quad = \frac{1}{2} \times (7x+6) \times 4x$$

$$h = 4x \quad = (7x+6) \times (2x)$$

$$= 14x^2 + 12x$$

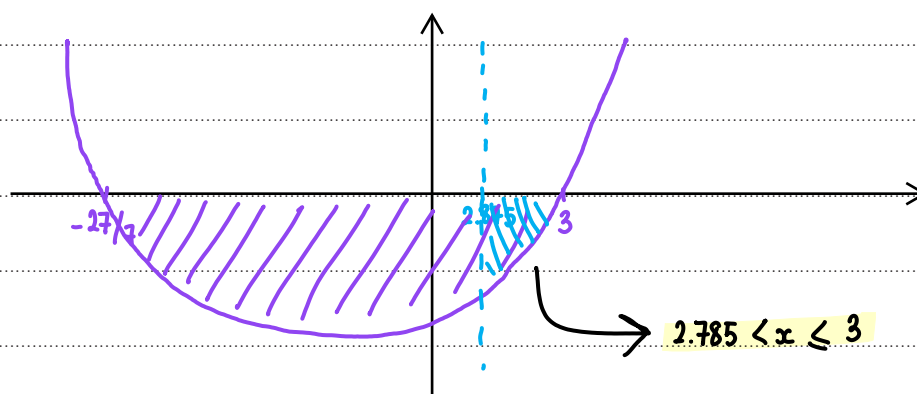
(c) You now need to form algebraic inequalities using your equations for perimeter and area.

$$\begin{aligned} 16x + 6 &> 52 \\ -6 & \quad -6 \\ \hline 16x &> 46 \\ \div 16 & \quad \div 16 \\ \hline x &> 2.875 \end{aligned}$$

$$\begin{aligned} 14x^2 + 12x &\leq 162 \\ -162 & \quad -162 \\ \hline 14x^2 + 12x - 162 &\leq 0 \\ \div 2 & \quad \div 2 \\ \hline 7x^2 + 6x - 81 &\leq 0 \\ -567 & \\ \hline (7x - 21)(7x + 27) &\leq 0 \\ \cancel{7} & \end{aligned}$$

$$(x-3)(7x+27) \leq 0$$

$$x=3 \quad x=-\frac{27}{7}$$

Now draw a diagram and determine values of x .

Question 8 continued

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$$\mathbf{p} = \begin{pmatrix} 2x-1 \\ y \end{pmatrix}$$

$$\mathbf{q} = \begin{pmatrix} y+3 \\ -y \end{pmatrix}$$

The vectors $\mathbf{p}$ and $\mathbf{q}$ are such that $|\mathbf{p}| = \sqrt{98}$ and $\mathbf{p} + \mathbf{q} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$

(a) Show that $x^2 - 3x - 9 = 0$

(5)

Given that $x > 0$

(b) (i) find the exact value of x ,

(2)

(ii) show that $y = 2 - 3\sqrt{5}$

(2)

(c) Find the exact value of $|\mathbf{q}|^2$
Show your working clearly.

(3)

1a) You can use the information given to form equations for p and q .

$$|\mathbf{p}| = \sqrt{98}$$

$$|\mathbf{p}| = \sqrt{(2x-1)^2 + y^2}$$

$$\sqrt{98} = \sqrt{(2x-1)^2 + y^2}$$

$$98 = (2x-1)^2 + y^2$$

$$\mathbf{p} + \mathbf{q} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2x-1 \\ y \end{pmatrix} + \begin{pmatrix} y+3 \\ -y \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$$

$$(2x-1) + (y+3) = 7$$

$$2x + y + 2 = 7$$

$$y = 5 - 2x$$

$$y + -y = 0$$

$$0 = 0$$

$$\left[\text{Solutions of } ax^2 + bx + c = 0 \text{ are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$



Question 9 continued

$$98 = (2x-1)^2 + y^2$$

$$98 = (2x-1)^2 + (5-2x)^2$$

$$98 = 4x^2 - 4x + 1 + (25 - 10x + 4x^2)$$

$$\begin{aligned} 98 &= 8x^2 - 14x + 26 \\ -98 & \quad -98 \\ 0 &= 8x^2 - 14x - 72 \\ \div 8 & \quad \div 8 \\ 0 &= x^2 - 3x - 9 \end{aligned}$$

1b) (i) Use the quadratic formula to find solutions for x .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$1x^2 - 3x - 9 = 0$$

where $a = 1$

$$x_1 = \frac{-(-3) + \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$

$b = -3$

$c = -9$

$$= \frac{3 + 3\sqrt{5}}{2}$$

$$x_2 = \frac{-(-3) - \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$

$$= \frac{3 - 3\sqrt{5}}{2} \rightarrow \text{This is less than 0 so reject}$$

$$x = \frac{3 + 3\sqrt{5}}{2}$$

(ii) You can now substitute your value of x into an equation for y .

$$y = 5 - 2x$$

$$y = 5 - 2\left(\frac{3 + 3\sqrt{5}}{2}\right)$$

$$y = 5 - (3 + 3\sqrt{5})$$

$$y = 2 - 3\sqrt{5}$$



Question 9 continued

(c) for $|q|^2$ you will need the square of the modulus of q .

$$q = \begin{pmatrix} y+3 \\ -y \end{pmatrix}$$

$$|q| = \sqrt{(y+3)^2 + (y)^2}$$

$$|q|^2 = (y+3)^2 + y^2$$

$$|q|^2 = (2-3\sqrt{5}+3)^2 + (2-3\sqrt{5})^2 \quad \leftarrow \text{Sub } y=2-3\sqrt{5}$$

$$|q|^2 = (5-3\sqrt{5})^2 + (2-3\sqrt{5})^2 \quad \leftarrow \text{Simplify}$$

$$= (25 - 30\sqrt{5} + 45) + (4 - 12\sqrt{5} + 45) \quad \leftarrow \text{Expand}$$

$$= 19 - 42\sqrt{5} \quad \leftarrow \text{Simplify}$$



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Question 9 continued

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(Total for Question 9 is 12 marks)



10 The curve C has equation $y = 2x^3 + ax^2 + bx + 10$ where a and b are constants.

The point A , with coordinates $(1, 3)$, lies on C .

(a) Write down an equation in a and b .

(1)

The tangent to C at the point A has gradient -8

(b) Show that $2a + b = -14$

(3)

(c) Hence find the value of a and the value of b .
Show clear algebraic working.

(3)

Using the value of a and the value of b found in part (c),

(d) find the x coordinate of each of the points on C where the tangent to C is parallel to the line with equation $y = 7 - 4x$.
Show clear algebraic working.

(3)

(a) Substitute $(1, 3)$ into $y = 2x^3 + ax^2 + bx + 10$

$$3 = 2(1)^3 + a(1)^2 + b(1) + 10$$

$$3 = 2 + a + b + 10$$

$$3 = a + b + 12$$

$$-9 = a + b$$

(b) A tangent to a curve has the same gradient as the curve at that point.

Differentiate y for gradient equation:

The general rule for differentiation is "bring down the power and subtract 1"

$$y = 2x^3 + ax^2 + bx + 10$$

$$\frac{dy}{dx} = (3)2x^{3-1} + (2)ax^{2-1} + b$$

$$\frac{dy}{dx} = 6x^2 + 2ax + b$$

Now substitute gradient $= -8$ and $x = 1$

$$(-8) = 6(1)^2 + 2a(1) + b$$

$$-8 = 6 + 2a + b$$

$$-14 = 2a + b$$



Question 10 continued

(c) You now have 2 simultaneous equations in terms of a and b which you can solve.

$$\textcircled{1} a + b = -9$$

$$\textcircled{2} 2a + b = -14$$

There are two ways to solve simultaneous equations:

A: Elimination

B: Substitution

$\textcircled{1}$ Since the value in front of b is the same we can eliminate b by subtraction.

$$\begin{array}{r} a + b = -9 \\ -2a + b = -14 \\ \hline -a = 5 \\ \times -1 \quad \times -1 \\ \hline a = -5 \end{array}$$

$\textcircled{2}$ You can now find b by substituting your value of a .

$$\begin{array}{r} (-5) + b = -9 \\ +5 \quad +5 \\ \hline b = -4 \end{array}$$

$\textcircled{1}$ Make a or b the subject of one of the equation

$$a + b = -9$$

$$a = -9 - b$$

$\textcircled{2}$ Substitute into the other equation

$$\begin{array}{r} 2(-9 - b) + b = -14 \\ -18 - 2b + b = -14 \\ \text{Simplify} \rightarrow -18 - b = -14 \\ +18 \quad +18 \\ \hline -b = 4 \\ \div -1 \quad \div -1 \\ \hline b = -4 \end{array}$$

$\textcircled{3}$ Substitute your value of b to find a

$$a = -9 - (-4)$$

$$a = -5$$

(d) The tangent to C which is parallel to line $y = 7 - 4x$ will have a gradient of -4 since parallel lines have the same gradient.

So if we can find the coordinates of the points on C with a gradient of -4 using the derivative, we can find their equation.

For this, we will use the differentiated equation for C and substitute the found values of a and b .

$$\frac{dy}{dx} = 6x^2 + 2ax + b$$



Question 10 continued

$$\frac{dy}{dx} = 6x^2 + 2ax + b$$

$$\frac{dy}{dx} = 6x^2 + 2(-5)x + (-4)$$

$$\frac{dy}{dx} = 6x^2 - 10x - 4$$

Now, you can substitute a gradient of -4 and find your two values for x .

$$\begin{aligned}
 -4 &= 6x^2 - 10x - 4 \\
 +4 &\quad \swarrow \quad \searrow +4 \\
 0 &= 6x^2 - 10x \\
 &\quad \searrow \text{factorise} \\
 0 &= 2x(3x - 5) \\
 \begin{aligned}
 2x &= 0 & 3x - 5 &= 0 \\
 \div 2 &\quad \swarrow \quad \searrow \div 2 & +5 &\quad \swarrow \quad \searrow +5 \\
 x &= 0 & 3x &= 5 \\
 & & \div 3 &\quad \swarrow \quad \searrow \div 3 \\
 & & x &= \frac{5}{3}
 \end{aligned}
 \end{aligned}$$



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Question 10 continued

Handwriting practice area with horizontal dotted lines.

(Total for Question 10 is 10 marks)



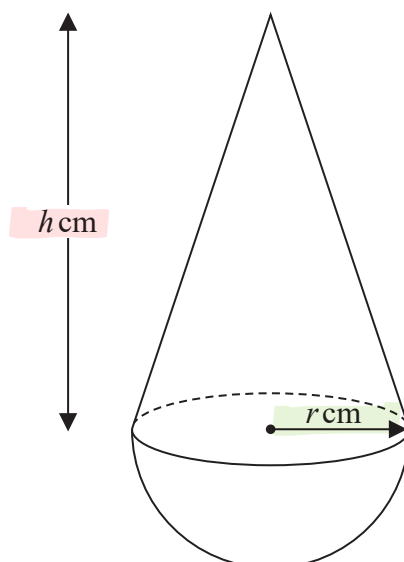


Diagram **NOT**
accurately drawn

Figure 4

A child's toy is made by fixing a solid right circular cone, with base radius r cm and height h cm, on the flat circular face of a solid hemisphere of radius r cm. The centre of the base of the cone coincides with the centre of the hemisphere, as shown in Figure 4

Given that $h + 6r = 15$

- (a) find the upper bound for the value of r .
Give a reason for your answer.

(2)

The volume of the toy is V cm³

- (b) Show that $V = \frac{1}{3}\pi r^2(15 - 4r)$

(3)

(a) To find the upper bound of r , we first need to make r the subject of the equation.

$$h + 6r = 15$$

$$6r = 15 - h$$

$$r = \frac{15 - h}{6}$$

Since we want the upper bound of r , h must be as small as possible so we can assume $h = 0$.

$$r = \frac{15 - (0)}{6}$$

$r < 2.5$ We use the $<$ sign since it is not inclusive.

$$\left[\begin{array}{l} \text{Volume of cone} = \frac{1}{3}\pi r^2 h \\ \text{Volume of sphere} = \frac{4}{3}\pi r^3 \end{array} \right]$$



Question 11 continued

(b) To find an expression for the total volume, you need to find the volume of the cone and sphere separately.

Volume of cone: $\frac{1}{3} \times \pi \times r^2 \times h = \frac{1}{3} \pi r^2 h$

Volume of sphere: $\frac{1}{2} \times \left(\frac{4}{3} \pi r^3 \right) = \frac{4}{6} \pi r^3 = \frac{2}{3} \pi r^3$

Total volume: $\frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3$

Now you need to rearrange the given equation $h + 6r = 15$ to make h the subject.

$$h + 6r = 15$$

$$h = 15 - 6r$$

$$\begin{aligned} \text{Total volume} &= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 \\ &= \frac{1}{3} \pi r^2 (15 - 6r) + \frac{2}{3} \pi r^3 \quad \text{Sub } h = 15 - 6r \\ &= 5\pi r^2 - 2\pi r^3 + \frac{2}{3} \pi r^3 \quad \text{Expand} \\ &= 5\pi r^2 - \frac{4}{3} \pi r^3 \quad \text{Simplify} \\ &= \frac{1}{3} \pi r^2 (15 - 4r) \quad \text{Take out a factor of } \frac{1}{3} \end{aligned}$$

Question 11 continues on the next page



Question 11 continued

- (c) Complete the following table of values for $V = \frac{1}{3}\pi r^2(15 - 4r)$

Give your values of V to one decimal place.

(3)

r	0	0.4	0.8	1.2	1.6	2.0	2.4
V	0	2.2	7.9	15.4	23.1	29.3	32.6

- (d) On the grid opposite, plot the points from your completed table and join them to form a smooth curve.

(2)

The volume of one particular toy is 26 cm^3

- (e) Use your curve to find, to one decimal place, the value of r .

(1)

The manufacturer of the toy decides that the value of V should be twice the value of h .

- (f) By drawing a suitable straight line on the grid, find an estimate, to the nearest integer, for the value of V .

(3)

(c) Substitute each value of r into the given equation for volume.

$$V = \frac{1}{3}\pi (0.4)^2 (15 - 4(0.4)) = 2.24\dots$$

$$\approx 2.2$$

$$V = \frac{1}{3}\pi (1.2)^2 (15 - 4(1.2)) = 15.38\dots$$

$$\approx 15.4$$

$$V = \frac{1}{3}\pi (1.6)^2 (15 - 4(1.6)) = 23.05\dots$$

$$\approx 23.1$$

(e) 2.0

(f) Since the value of V is twice that of h , we can multiply the equation of h for V .

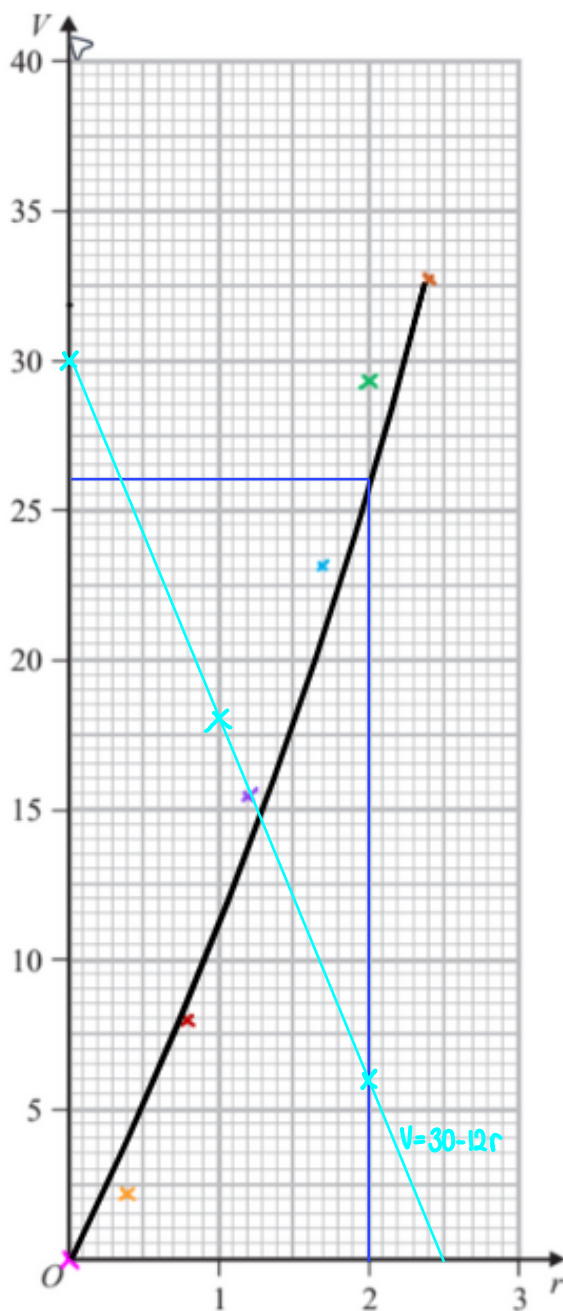
$$h = 15 - 6r$$

$$V = 30 - 12r$$

You can now plot this equation for a line with gradient -12 and y -intercept of 30 .



Question 11 continued

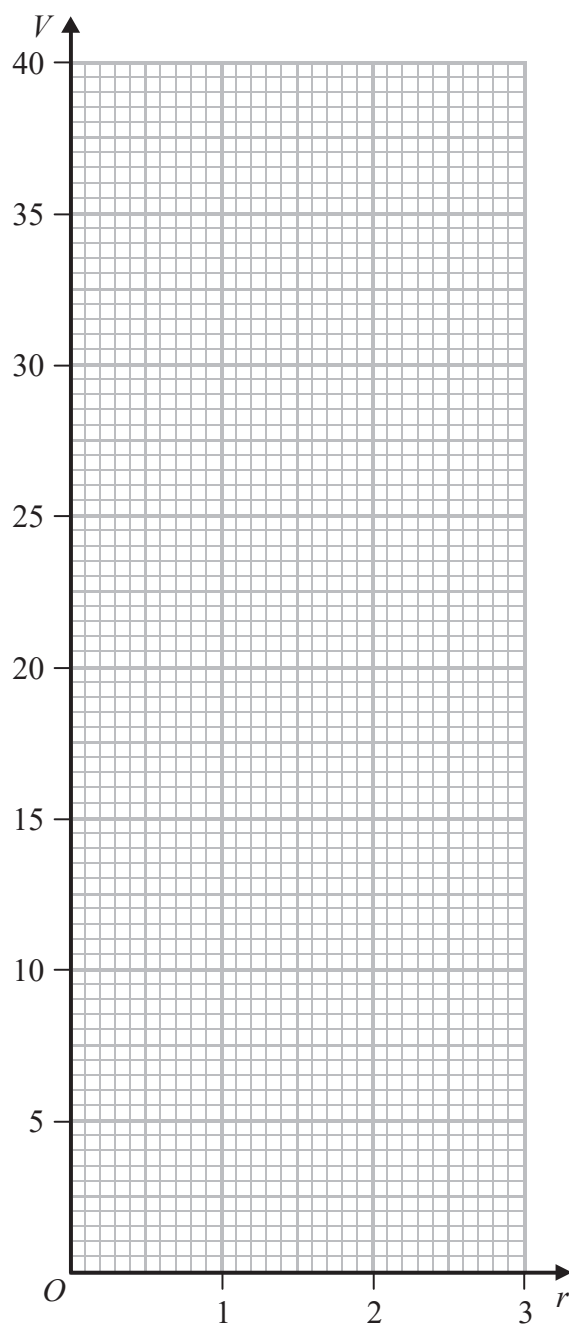


Turn over for a spare grid if you need to redraw your curve.



Question 11 continued

Only use this grid if you need to redraw your curve.



(Total for Question 11 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS

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